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Natural convection flow of a viscous fluid about a truncated cone with temperature-dependent viscosity and thermal conductivity

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Abstract *A steady, two-dimensional natural convection flow of a viscous, incompressible fluid having temperature-dependent viscosity and thermal conductivity about a truncated cone is considered. We use suitable transformations to obtain the equations governing the flow in convenient form and integrate them by using an implicit finite difference method. Perturbation solutions are employed to obtain the solution in the regimes near and far away from the point of truncation. The results are obtained in terms of the local skin friction and the local Nusselt number. Perturbation solutions are compared with the finite difference solutions and found to be in excellent agreement. The dimensionless velocity, viscosity and thermal conductivity distributions are also displayed graphically, showing the effects of various values of the pertinent parameter for smaller values of Prandtl number.*

Nomenclature

x = streamwise coordinate
x^* = distance measured from the leading edge of the truncated cone
x_0 = distance of the leading edge of the truncated cone measured from the origin
y = transverse coordinate
u = velocity component in the x-direction
v = velocity component in the y-direction
f = dimensionless stream function
g = acceleration due to gravity
T = temperature of the fluid
T_∞ = temperature of the ambient fluid
T_w = surface temperature
C_p = specific heat
Pr = Prandtl number
Gr_x = local Grashof number
ψ = stream function
θ = dimensionless temperature function
ξ = dimensionless distance
η = similarity variable
β = coefficient of thermal expansion
ν = kinematic viscosity
ρ = fluid density
μ = viscosity of the fluid
μ_∞ = viscosity of the ambient fluid at infinity
ε = viscosity variation parameter
κ = thermal conductivity
κ_∞ = thermal conductivity of the ambient fluid
γ = thermal conductivity variation parameter
Nu_x = Nusselt number
C_f = skin friction

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Introduction

Na and Chiou (1979a, 1980) studied the effect of slenderness on the natural convection flow of viscous, incompressible fluid over the frustum of a cone. For an isothermal surface, the problem of natural convection flow over the frustum of a cone without transverse curvature effect was treated by Na (1979b). Hering and Grosh (1962) investigated the laminar natural convection from a non-isothermal cone and showed that similarity solutions exist when the cone wall temperature varies as a power of distance along a cone ray. Later, Hering (1965) extended the analysis given in Hering and Grosh (1962) by investigating the problem for the fluids of low Prandtl numbers. For the case of high Prandtl numbers, the problem posed by Hering and Grosh (1962) had been extended by Roy (1974). Alamgir (1979) studied the overall heat transfer from vertical cones in laminar natural convection for all Prandtl numbers.

In all the above studies the various authors assumed that the fluids have uniform viscosity and thermal conductivity throughout the flow regime. However, it is known that this physical property may change significantly with temperature. When the effect of viscosity is included in the analysis, Gary *et al.* (1982) and Mehta and Sood (1992) have found that the flow characteristics substantially change compared with the constant viscosity cases. Mindful of this, Hady *et al.* (1996), Kafoussias and Williams (1997) and Kafoussias and Rees (1998) investigated the effect of temperature-dependent viscosity on the mixed convection flow from a vertical flat plate in the region near the leading edge. Very recently, Hossain *et al.* (2000a, 2001) have investigated the natural convection flow from a vertical wavy surface and a truncated cone respectively. The problems of mixed convection flow from a vertical plate and that of a forced convection flow past a wedge for temperature-dependent viscosity have been investigated by Hossain *et al.* (2000b) and Hossain and Munir (2000). In all the above studies (Hossain *et al.*, 2000a, 2000b; Hossain and Munir, 2000) the viscosity of the fluid has been considered to be inversely proportional to a linear function of temperature. On the other hand, for a liquid it has been found that thermal conductivity κ varies with temperature in an approximately linear manner in the range from 0°F to 400°F (see Kays, 1966). A semi-empirical formula for thermal conductivity was used by Arunachalam *et al.* (1978). Following these authors, Chaim (1998) has investigated the effect of variable thermal conductivity over a linearly stretching sheet. The viscosity and thermal conductivity of the fluid have been assumed to be proportional to a linear function of temperature, two semi-empirical formulae for the viscosity and thermal conductivity as proposed by Charraudeau (1975). Recently, Hossain *et al.* (2000c) have investigated the flow past a wedge for a fluid having temperature-dependent viscosity and thermal conductivity.

Here, it is proposed to investigate the natural convection flow of a viscous, incompressible fluid from an isothermal truncated cone. In formulating the equations governing the flow under consideration, it has been assumed that the fluid property variations are limited to viscosity and to the thermal conductivity on temperature of the fluid in the boundary layer region. The

Boussinesq approximation is also assumed in formulating the buoyancy term. The governing partial differential equations are reduced to locally non-similar equations by introducing the transformations appropriate for the natural convection flow over a slender circular cylinder (Sparrow and Gregg, 1972), the solutions of which are obtained by using an implicit finite difference method. Asymptotic solutions are sought in the regimes near the point of truncation and far from this point.

Effects of the viscosity variation parameter, ε, and the thermal conductivity variation parameter, γ, on the local skin-friction and the local Nusselt number are depicted in tabular form, having Pr = 0.01, 0.02, 0.05 and 0.07. The velocity, viscosity and thermal conductivity distributions are shown graphically for fluids having Pr = 0.01 only.

Mathematical formalism

We consider the steady, two-dimensional laminar natural convection flow of a viscous incompressible fluid about a truncated cone. Figure 1 shows the flow model and physical coordinate system. The origin of the coordinate system is placed at the vertex of the full cone, where x and y are the coordinate along and normal to the surface of cone, respectively, measured from the origin. The boundary layer is assumed to develop at the leading edge of the cone ($x = x_0$), which implies that the temperature at the circular base is assumed to be the same as the ambient temperature T_∞. The temperature of the truncated cone T_w is uniform and higher than the free stream temperature T_∞. As stated in the introduction, property variations with temperature are limited to density, viscosity and thermal conductivity. Variations in density are taken into account only in so far as its effects on the buoyancy term in the momentum equation are concerned (Boussinesq approximation).

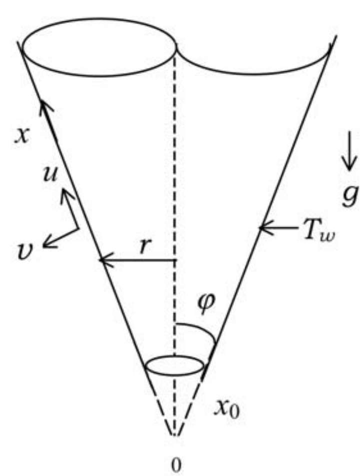


Figure 1.
The geometry and the coordinate system

Under the above assumptions, the two-dimensional boundary layer equations for natural convective flow of the fluid over a truncated cone are as follows:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta\cos\varphi(T - T_\infty) + \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right), \tag{3}$$

where u, v are fluid velocity components in the x- and y-direction respectively, g is the gravitational acceleration, β is the coefficient of thermal expansion, T is the temperature inside the boundary layer and α is the thermal diffusivity.

The boundary conditions which satisfy the above equations are as follows:

$$\begin{aligned} &u = 0,\ v = 0,\ T = T_w \text{ at } y = 0,\\ &u = 0,\ T = T_\infty \text{ as } y \to \infty. \end{aligned} \tag{4}$$

In the present investigation, we use two semi-empirical formulae for viscosity and thermal conductivity, which introduced by Charraudeau (1975), as follows:

$$\begin{aligned} &\mu = \mu_f\left[1 + \frac{1}{\mu_f}\left(\frac{d\mu}{dT}\right)_f(T - T_\infty)\right] \text{ and}\\ &\kappa = \kappa_f\left[1 + \frac{1}{\kappa_f}\left[\left(\frac{d\kappa}{dT}\right)_f(T - T_\infty)\right]\right], \end{aligned} \tag{5}$$

where the suffix "f" denotes the quantities outside the boundary layers.

Transformations and methods of solution

We have assumed the boundary layer to be sufficiently thin in comparison with the local radius of the truncated cone. The local radius to a point in the boundary layer can be replaced by the radius of the truncated cone r, $r = x\sin\varphi$. Equations (1)-(4) are valid in $x_0 \le x < \infty$.

Now we define the following dimensionless variables:

$$\psi = \nu_f r Gr_{x*}^{1/4} f(\xi,\eta), \eta = \frac{y}{x*}Gr_{x*}^{1/4}, \xi = \frac{x*}{x_0} = \frac{x - x_0}{x_0}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \tag{6}$$

where ψ is the stream function that satisfies the continuity equation defined by:

$$ru = \frac{\partial \psi}{\partial y} \text{ and } rv = -\frac{\partial \psi}{\partial x}, \tag{7}$$

and Gr_x is the local Grashof number, defined by:

$$Gr_{x*} = g\beta \cos\varphi(T_w - T_\infty)x^{*3}/\nu_f^2, \tag{8}$$

where ν_f (= μ_f/ρ) is the free stream kinematic viscosity, $f(\xi, \eta)$ is the dimensionless stream function, η is pseudo similarity variable and θ is the dimensionless temperature of the fluid in the boundary layer region.

Substitution of the transformation given by equation (6) into equations (1)-(4) gives the following non-similar equations governing flow and energy distribution:

$$(1 + \varepsilon\theta)f''' + \varepsilon\theta' f'' + \left(\frac{3}{4} + \frac{\xi}{1+\xi}\right)ff'' - \frac{1}{2}f'^2 + \theta = \xi\left(f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right), \tag{9}$$

$$(1 + \gamma\theta)\theta'' + \gamma\theta'^2 + \Pr\left(\frac{3}{4} + \frac{\xi}{1+\xi}\right)f\theta' = \Pr\xi\left(f'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial f}{\partial \xi}\right), \tag{10}$$

where, ε and γ are, respectively, the viscosity-variation parameter and the thermal conductivity variation parameter, which are defined by:

$$\varepsilon = \frac{1}{\mu_f}\left(\frac{d\mu}{dT}\right)_f (T_w - T_\infty) \text{ and } \gamma = \frac{1}{\kappa_f}\left(\frac{d\kappa}{dT}\right)_f (T_w - T_\infty).$$

The corresponding boundary conditions are:

$$\begin{aligned} &f(\xi, 0) = f'(\xi, 0) = 0, \theta(\xi, 0) = 1, \\ &f'(\xi, \infty) = 0, \theta(\xi, \infty) = 0. \end{aligned} \tag{11}$$

We propose to simulate the system of equations (9)-(11) by an implicit finite difference method, which has, recently, been used most efficiently by Hossain *et al.* (2000a, 2000b, 2000c, 2001) and Hossain and Munir (2000). According to this method, the system of partial differential equations considered here is first converted to a system of five first-order partial differential equations by introducing new functions of the η derivatives. This system is transformed into the finite-difference scheme in which the resulting non-linear differential equations are linearized by using Newton's quasi-linearization method. The resulting linear differential equations, along with the boundary conditions, are finally solved by an efficient block-tridiagonal factorization method introduced by Keller (1978).

Solution near the truncated point

For small ξ, the ratio $\xi/[1+\xi] \rightarrow \xi$, and equations (9) and (10) reduce to:

$$(1+\varepsilon\theta)f''' + \varepsilon\theta' f'' + \left(\frac{3}{4}+\xi\right)ff'' - \frac{1}{2}f'^2 + \theta = \xi\left(f'\frac{\partial f'}{\partial\xi} - f''\frac{\partial f}{\partial\xi}\right), \quad (12)$$

$$(1+\gamma\theta)\theta'' + \gamma\theta'^2 + \Pr\left(\frac{3}{4}+\xi\right)f\theta' = \Pr\xi\left(f'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial f}{\partial\xi}\right), \quad (13)$$

subject to the boundary conditions:

$$\begin{aligned} &f(\xi,0) = f'(\xi,0) = 0,\ \theta(\xi,0) = 1, \\ &f'(\xi,\infty) = 0, \theta(\xi,\infty) = 0. \end{aligned} \quad (14)$$

Since ξ is small, solutions to the equations (12)-(14) may be obtained by using the perturbation method. We expand the functions $f(\xi,\eta)$ and $\theta(\xi,\eta)$ in powers of ξ, as given below:

$$f(\xi,\eta) = \sum_{i=0}^{\infty}\xi^i f_i(\eta) \text{ and } \theta(\xi,\eta) = \sum_{i=0}^{\infty}\xi^i\theta_i(\eta). \quad (15)$$

Now, substituting the above expansions in equations (12)-(14) and taking the terms only up to $O(\xi^2)$ we get:

$$(1+\varepsilon\theta_0)f_0''' + \varepsilon\theta_0' f_0'' + \frac{3}{4}f_0 f_0'' - \frac{1}{2}f_0'^2 + \theta_0 = 0, \quad (16)$$

$$(1+\gamma\theta_0)\theta_0'' + \gamma\theta_0'^2 + \Pr\frac{3}{4}f_0\theta_0' = 0, \quad (17)$$

$$\begin{aligned} &f_0(0) = f_0'(0) = 0,\ \theta_0(0) = 1 \\ &f_0'(\infty) = 0,\ \theta_0(\infty) = 0, \end{aligned} \quad (18)$$

$$\begin{aligned} &(1+\varepsilon\theta_0)f_1''' + \varepsilon\left(\theta_1 f_0''' + \theta_0' f_1'' + \theta_1' f_0''\right) + \frac{3}{4}f_0 f_1'' + \frac{7}{4}f_1 f_0'' + f_0 f_0'' \\ &\quad + \theta_1 - 2f_1' f_0' = 0, \end{aligned} \quad (19)$$

$$\begin{aligned} &(1+\gamma\theta_0)\theta_1'' + \gamma\left(\theta_1\theta_0'' + 2\theta_0'\theta_1'\right) \\ &+ \Pr\left(\frac{3}{4}f_0\theta_1' + \frac{7}{4}f_1\theta_0' + f_0\theta_0' - \theta_1 f_0'\right) = 0, \end{aligned} \quad (20)$$

$$
\begin{aligned}
&f_1(0) = f_1'(0) = 0,\ \theta_1(0) = 0 \\
&f_1'(\infty) = 0,\ \theta_1(\infty) = 0,
\end{aligned}
\tag{21}
$$

$$
\begin{gathered}
(1+\varepsilon\theta_0)f_2''' + \varepsilon\left(\theta_1 f_1''' + \theta_2 f_0''' + \theta_0' f_2'' + \theta_1' f_1'' + \theta_0' f_2''\right) + \frac{3}{4}f_0 f_2'' + \frac{7}{4}f_1 f_1'' \\
+\frac{11}{4}f_2 f_0'' + f_1 f_0'' + f_0 f_1'' + \theta_2 - 3f_2' f_0' - \frac{3}{2}f_1'^2 = 0,
\end{gathered}
\tag{22}
$$

$$
\begin{gathered}
(1+\gamma\theta_0)\theta_2'' + \gamma\left(\theta_1\theta_1'' + \theta_2\theta_0'' + 2\theta_0'\theta_2' + \theta_1'^2\right) \\
+\Pr\left(\frac{3}{4}f_0\theta_2' + \frac{7}{4}f_1\theta_1' + \frac{11}{4}f_2\theta_0' + f_1\theta_0' + f_0\theta_1' - 2\theta_2 f_0' - \theta_1 f_1'\right) = 0,
\end{gathered}
\tag{23}
$$

$$
\begin{aligned}
&f_2(0) = f_2'(0) = 0,\ \theta_2(0) = 0 \\
&f_2'(\infty) = 0,\ \theta_2(\infty) = 0.
\end{aligned}
\tag{24}
$$

Since equations (16) and (17) are coupled and non-linear, so the solutions of these equations can be obtained by the Nachtsheim-Swigert iteration technique together with the sixth order implicit Runge-Kutta-Butcher initial value solver. Solutions of the subsequent sets of equations are also obtained by employing the aforementioned iterative technique.

It should be noted that equations (16) and (17) represent the similarity equations for the free convection flow of a fluid of variable viscosity and thermal conductivity from a vertical isothermal flat plate.

After knowing the values of the functions f and θ and their derivatives we can calculate the values of the local friction coefficient and the local Nusselt number in the region near the point of truncation from the following relations:

$$
\frac{(1+\varepsilon)}{2} C_f Gr_{x*}^{1/4} = f_0''(0) \ + \xi f_1''(0) + \xi^2 f_2''(0)\cdots, \tag{25a}
$$

and:

$$
\frac{Nu_{x*}}{Gr_{x*}^{1/4}} = -\left[\theta_0'\ (0) + \xi\theta_1'\ (0) + \xi^2\theta_2'\ (0)\cdots\right]. \tag{25b}
$$

Solution at a distance from the truncated point

In the region far away from the point of truncation ξ becomes very large and hence we may introduce a new variable, ζ, depending on ξ, which is defined by $\zeta = 1/\xi$. Introducing this new variable into equations (9)-(11), one obtains:

$$(1+\varepsilon\theta)f''' + \varepsilon\theta' f'' + \left[\left(\frac{3}{4}+\frac{1}{1+\zeta}\right)ff'' - \frac{1}{2}f'^2 + \theta\right] = \zeta\left(f''\frac{\partial f}{\partial\zeta} - f'\frac{\partial f'}{\partial\zeta}\right), \tag{26}$$

$$(1+\gamma\theta)\theta'' + \gamma\theta'^2 + \left(\frac{3}{4}+\frac{1}{1+\zeta}\right)\mathrm{Pr}\, f\theta' = \mathrm{Pr}\,\zeta\left(\theta'\frac{\partial f}{\partial\zeta} - f'\frac{\partial\theta}{\partial\zeta}\right), \tag{27}$$

subject to the boundary conditions:

$$\begin{aligned} &f(\zeta,0) = f'(\zeta,0) = 0,\ \theta(\zeta,0) = 1 \\ &f'(\zeta,\infty) = 0,\ \theta(\zeta,\infty) = 0. \end{aligned} \tag{28}$$

Since ζ is small we may approximate $1/(1+\zeta) \approx 1-\zeta$, and so we can write equations (26)-(28) to the following form:

$$(1+\varepsilon\theta)f''' + \varepsilon\theta' f'' + \left[\left(\frac{7}{4}-\zeta\right)ff'' - \frac{1}{2}f'^2 + \theta\right] = \zeta\left(f''\frac{\partial f}{\partial\zeta} - f'\frac{\partial f'}{\partial\zeta}\right), \tag{29}$$

$$(1+\gamma\theta)\theta'' + \gamma\theta'^2 + \left(\frac{7}{4}-\zeta\right)\mathrm{Pr}\, f\theta' = \mathrm{Pr}\,\zeta\left(\theta'\frac{\partial f}{\partial\zeta} - f'\frac{\partial\theta}{\partial\zeta}\right). \tag{30}$$

The boundary conditions are the same as in equation (28).

Now we can expand the functions f(ζ,η) and $\theta(\zeta,\eta)$ in powers of ζ, as given below:

$$f(\zeta,\eta) = \sum_{i=0}^{\infty}(\zeta)^i f_i(\eta) \text{ and } \theta(\zeta,\eta) = \sum_{i=0}^{\infty}(\zeta)^i\theta_i(\eta). \tag{31}$$

Now, substituting the above expansions into equations (29) and (30) and taking the terms only up to $O(\zeta)$, we get:

$$(1+\varepsilon\theta_0)f_0''' + \varepsilon\theta_0' f_0'' + \frac{7}{4}f_0 f_0'' - \frac{1}{2}f_0'^2 + \theta_0 = 0, \tag{32}$$

$$(1+\gamma\theta_0)\theta_0'' + \gamma\theta_0'^2 + \mathrm{Pr}\frac{7}{4}f_0\theta_0' = 0, \tag{33}$$

$$\begin{aligned} &f_0(0) = f_0'(0) = 0,\ \theta_0(0) = 1 \\ &f_0'(\infty) = 0,\ \theta_0(\infty) = 0, \end{aligned} \tag{34}$$

$$(1+\varepsilon\theta_0)f_1''' + \varepsilon\left(\theta_1 f_0''' + \theta_0' f_1'' + \theta_1' f_0''\right) + \frac{7}{4} f_0 f_1'' + \frac{11}{4} f_1 f_0'' \\ +\theta_1 - 2f_1' f_0' = f_0 f_0'', \tag{35}$$

$$(1+\gamma\theta_0)\theta_1'' + \gamma\left(\theta_1\theta_0'' + 2\theta_0'\theta_1'\right) \\ +\Pr\left(\frac{7}{4} f_0\theta_1' + \frac{11}{4} f_1\theta_0' - \theta_1 f_0'\right) = \Pr f_0\theta_0', \tag{36}$$

$$f_1(0) = f_1'(0) = 0,\ \theta_1(0) = 0 \\ f_1'(\infty) = 0,\ \theta_1(\infty) = 0. \tag{37}$$

Equations (32)-(34) represent the similarity equations for the free convection flow of fluid with variable viscosity from a full cone. Since solutions to these equations with the effect of variable viscosity and thermal conductivity are not available in the literature, we present some solutions in Tables I and II. In these tables we also compare the results of Na and Chiou (1979b) for the case of a fluid with constant viscosity and thermal conductivity (i.e. $\varepsilon = \gamma = 0$) for different Pr.

As before, we solve the above sets of equations (32)-(37), and knowing the values of the functions f and θ and their derivatives we can calculate the asymptotic values of the local friction coefficient and the local Nusselt number from the following relations:

$$\frac{(1+\varepsilon)}{2} C_f Gr_{x*}^{1/4} = f_0''(0) + \xi^{-1} f_1''(0) + \cdots \text{ and} \\ \frac{Nu_{x*}}{Gr_{x*}^{1/4}} = -\left[\theta_0'(0) + \xi^{-1}\theta_1'(0) + \cdots\right]. \tag{38}$$

The resulting solutions for different pertinent parameters are entered and compared in Tables I and II with the finite difference solutions.

Results and discussion

We have investigated the problem of natural convection flow of a viscous, incompressible fluid with variable viscosity and thermal conductivity past a truncated cone. Perturbation solutions of the local non-similarity equations governing the flow are obtained in the region near the point of truncation and the asymptotic solution in the regime far away from the truncated point. For the entire regime starting from the leading edge to down stream governed by the equations (9)-(10) are obtained the implicit finite difference method. Overall solutions are obtained for fluids having Prandtl number, Pr = 0.01, appropriate for liquid metals, for $\xi \in (0, \infty)$ and for various values of the viscosity parameter ε = 0.0, 1, 2.5, 5.0, 10.0 and thermal conductivity parameter, γ = 0.0, 1, 2.5 and 5.0.

Table I.
Value of the local skin friction coefficient for different ξ at $\varepsilon = 0.0$, 1.0 and $\gamma = 0.0$, 1.0 while Pr = 0.01

	$\varepsilon = 0.0$				$\varepsilon = 1.0$			
	$\gamma = 0.0$		$\gamma = 1.0$		$\gamma = 0.0$		$\gamma = 1.0$	
ξ	Small and large ξ	Finite difference	Small and large ξ	Finite difference	Small and large ξ	Finite difference	Small and large ξ	Finite difference
0.00	1.39488[a]	1.39487	1.42823[a]	1.42822	0.95004[a]	0.95004	0.98238[a]	0.98237
0.25	1.41279[a]	1.40808	1.44614[a]	1.44534	0.96229[a]	0.95580	0.99484[a]	0.99125
0.75	1.44860[a]	1.41794	1.48196[a]	1.46068	0.98680[a]	0.95793	1.01976[a]	0.99745
1.75	1.53187[b]	1.41719	1.59277[b]	1.46573	1.01891[b]	0.95249	1.07274[b]	0.99586
3.75	1.43594[b]	1.40357	1.49304[b]	1.45616	0.95684[b]	0.93965	1.00615[b]	0.98523
7.75	1.39262[b]	1.38525	1.44800[b]	1.43917	0.92881[b]	0.92576	0.97608[b]	0.97171
15.75	1.37199[b]	1.37190	1.42655[b]	1.42591	0.91547[b]	0.91630	0.96176[b]	0.96207
31.75	1.36192[b]	1.36332	1.41608[b]	1.41711	0.90895[b]	0.91043	0.95477[b]	0.95593
63.75	1.35694[b]	1.35853	1.41090[b]	1.41211	0.90573[b]	0.90724	0.95132[b]	0.95253
∞	1.35200[b]	1.35199	1.40057[b]	1.39857	0.90059[b]	0.89395	0.94589[b]	0.94229

Notes: [a] small ξ; [b] large ξ

Table II.
Value of the rate of heat transfer for different ξ at $\varepsilon = 0.0$, 1.0 and $\gamma = 0.0$, 1.0 while Pr = 0.01

		$\varepsilon = 0.0$				$\varepsilon = 1.0$			
		$\gamma = 0.0$		$\gamma = 1.0$		$\gamma = 0.0$		$\gamma = 1.0$	
ξ	Na and Chiou (1979b)	Small and large ξ	Finite difference	Small and large ξ	Finite difference	Small and large ξ	Finite difference	Small and large ξ	Finite difference
0.00	0.05742	0.05748[a]	0.05748	0.03738[a]	0.03738	0.05637[a]	0.05637	0.03678[a]	0.03678
0.25	0.06135	0.06150[a]	0.06077	0.04139[a]	0.03951	0.06031[a]	0.05954	0.03933[a]	0.03886
0.75	0.06536	0.06953[a]	0.06467	0.04942[a]	0.04203	0.06820[a]	0.06331	0.04441[a]	0.04133
1.75	0.06895	0.06483[b]	0.06834	0.04216[b]	0.04441	0.06367[b]	0.06687	0.04150[b]	0.04366
3.75	0.07148	0.07021[b]	0.07104	0.04565[b]	0.04617	0.06881[b]	0.06950	0.04491[b]	0.04539
7.75	0.07307	0.07265[b]	0.07271	0.04722[b]	0.04727	0.07113[b]	0.07112	0.04644[b]	0.04647
15.75	0.07397	0.07381[b]	0.07371	0.04797[b]	0.04792	0.07223[b]	0.07210	0.04717[b]	0.04711
31.75	0.07444	0.07437[b]	0.07422	0.04834[b]	0.04826	0.07277[b]	0.07261	0.04753[b]	0.04744
63.75	0.07468	0.07465[b]	0.07448	0.04852[b]	0.04842	0.07304[b]	0.07286	0.04771[b]	0.04761
∞	0.07493	0.07493[b]	0.07476	0.04961[b]	0.04964	0.07452[b]	0.07516	0.04808[b]	0.04873

Notes: [a] small ξ; [b] large ξ

The numerical values of the local friction coefficient $(1+\varepsilon)C_f Gr_{x^*}^{1/4}/2$ and the local Nusselt number, $Nu_{x^*}/Gr_{x^*}^{1/4}$, for various $\xi \in (0, \infty)$ for a fluid having Pr = 0.01 while ε = 0.0 and 1.0 and γ = 0.0 and 1.0 are given in Tables I and II. In these tables, we also compare the finite difference solutions obtained for the entire regime with the perturbation solutions valid in the leading edge regime as well as in the regime far away from the leading edge. Comparison shows that the value of local Nusselt number obtained from the present analysis are in excellent agreement with those of Na and Chiou (1979b). From these tables we further observe that in the entire regime the local friction coefficient reaches some maximum value near the leading edge and then reaches the asymptotic value as ξ increases. On the other hand, the value of the local Nusselt number increases due to an increase in the value of the ξ. We further observe that an increase in the value of the viscosity variation parameter, ε, leads to a decrease in the value of the local friction coefficient as well as the local Nusselt number at every value of ξ. Similarly, we observe that when the value of thermal conductivity variation parameter, γ, increases there is an increase in the local skin friction coefficient and decrease in the Nusselt number.

In Table III we depict the value of the local skin friction coefficient $(1+\varepsilon)C_f Gr_{x^*}^{1/4}/2$ and the local Nusselt number, $Nu_{x^*}/Gr_{x^*}^{1/4}$ for different ξ at Pr = 0.002, 0.005, 0.015 and 0.02 while $\varepsilon = \gamma = 1$. From these figures we can observe that if the value of Pr increases then the value of local skin friction decreases, and that of local Nusselt number increases for all ξ.

Now we discuss the effect of the viscosity variation parameter, ε, and thermal conductivity variation parameter, γ, on the dimensionless velocity, dimensionless viscosity and the dimensionless thermal conductivity distributions obtained from the finite difference solutions of equations (9) and (10) governing the natural convection flow from the truncate cone. The dimensionless velocity, viscosity and thermal conductivity profiles are calculated from the following relations:

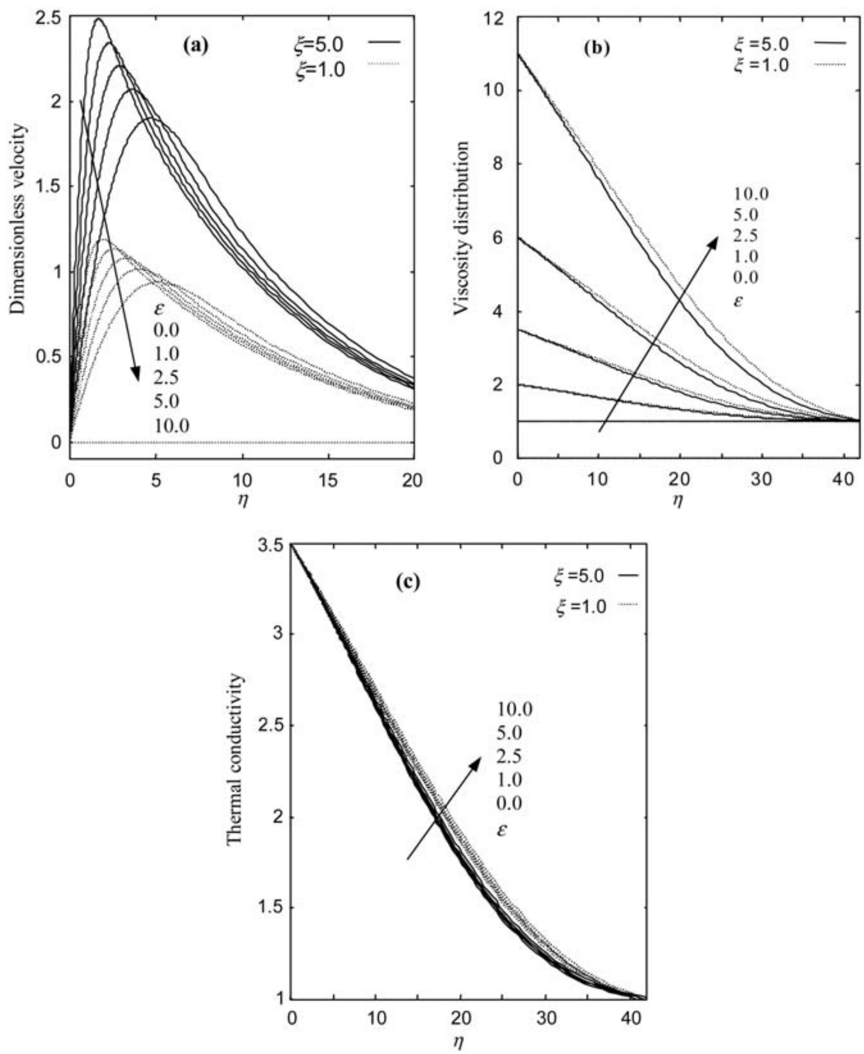
		$F''(\xi,0)$			$\theta(\xi,0)$	
ξ	Pr = 0.02	Pr = 0.05	Pr = 0.07	Pr = 0.02	Pr = 0.05	Pr = 0.07
0.0	0.94852	0.69632	0.66192	0.05171	0.19823	0.22409
0.1	0.95110	0.69133	0.65651	0.05293	0.20256	0.22885
0.2	0.95285	0.68668	0.65154	0.05396	0.20627	0.23294
0.4	0.95454	0.67835	0.64280	0.05562	0.21228	0.23958
0.6	0.95473	0.67122	0.63545	0.05688	0.21694	0.24474
0.8	0.95404	0.66511	0.62922	0.05787	0.22065	0.24886
1.0	0.95285	0.65985	0.62391	0.05867	0.22367	0.25223
5.0	0.92749	0.62051	0.58532	0.06360	0.24298	0.27387
10	0.91568	0.61001	0.57527	0.06475	0.24760	0.27907
10^2	0.90039	0.59821	0.56407	0.06599	0.25263	0.28475
10^3	0.89858	0.59690	0.56283	0.06613	0.25319	0.28539

Table III Value of the local skin friction coefficient and the rate of heat transfer for different ξ and Pr at ε = 1.0 and γ =1.0

$$u = \xi^{1/2} f'(\xi, \eta), \quad \frac{\mu}{\mu_f} = 1 + \varepsilon\theta(\xi, \eta) \text{ and } \frac{\kappa}{\kappa_f} = 1 + \gamma\theta\ (\xi, \eta), \tag{39}$$

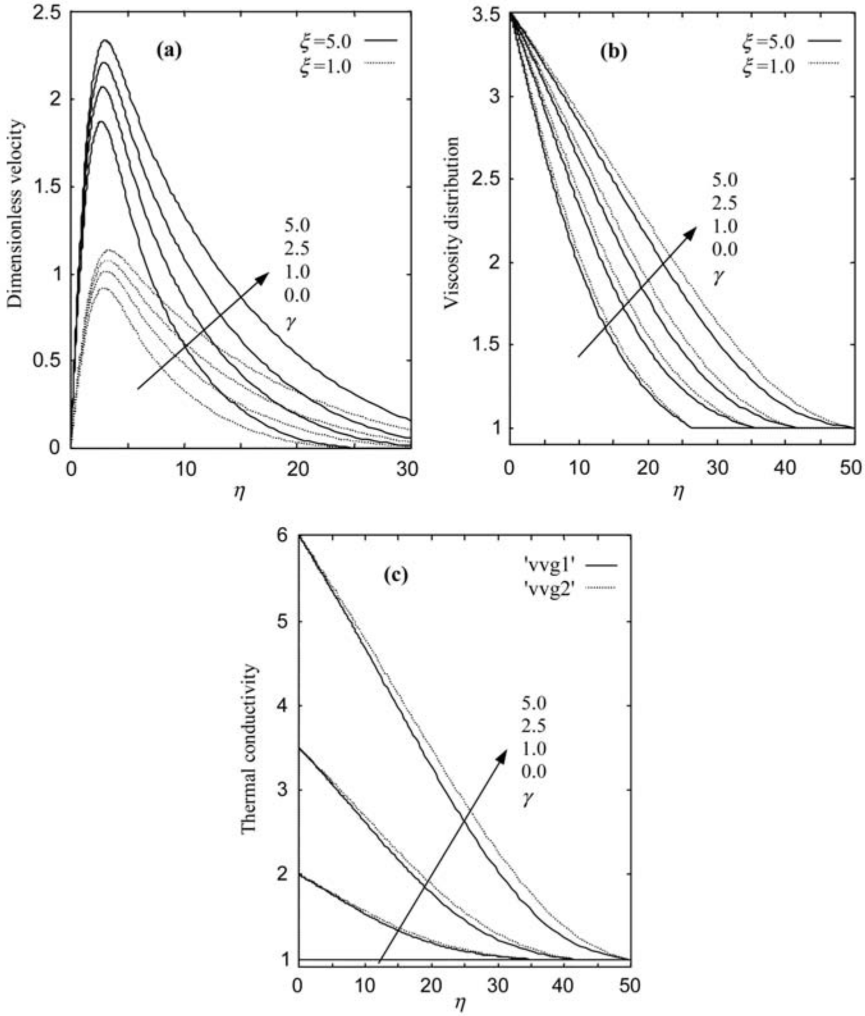
and are depicted in Figures 2 and 3. In these figures, the dotted and solid curves represent the above profiles for values of ξ =1.0 and 5.0 for the fluid having Prandtl number Pr = 0.01.

Figure 2(a) depicts the velocity profile for different values of ε (= 0.0, 1.0, 2.5, 5.0, 10.0) while the value of thermal conductivity variation parameter, γ, equals



Note: ε = 0.0, 1.0, 2.5, 5.0, 10.0 at γ = 2.5 and Pr = 0.01 while ξ = 1.0, 5.0

Figure 2.
Value of the (a) dimensionless velocity; (b) viscosity; (c) thermal conductivity



Note: γ = 0.0, 1.0, 2.5, 5.0 at ε = 2.5 and Pr = 0.01 while ξ = 1.0, 5.0

Figure 3.
Value of the dimensionless (a) velocity; (b) viscosity; (c) thermal conductivity

2.5. Corresponding profiles for the viscosity and thermal conductivity distributions for the fluid under consideration are shown in Figures 2(b) and 2(c), respectively. From Figure 2(a) it can be seen that an increase in the value of ε leads to a decrease in the velocity profile near the surface. It may further be noticed from Figures 2(b) and 2(c) that both the viscosity and thermal conductivity of the fluid increase owing to an increase in the value of ε at every station of ξ. But for the thermal conductivity the rate of increase due to increases in the viscosity parameter is less effective. For the interest of the experimentalist, we give the percentage of increasing or decreasing the values of the above-mentioned physical quantities. At ξ =1.0, while γ = 2.5 and

η =2.03686 the value of velocity decreases by 7.42 per cent, 19.06 per cent, 32.70 per cent, 47.89 per cent, when ε is increased from 0.0 to 1.0, 2.5, 5.0 and 10.0. A corresponding percentage increase in the viscosity and thermal conductivity are 93.61 per cent, 334 per cent, 569 per cent, 1,039 per cent and 0.07 per cent, 0.13 per cent, 0.21 per cent, 0.33 per cent, respectively.

The effect of an increase in the thermal conductivity variation parameter, γ (0.0, 1.0, 2.5 5.0), on the velocity profile at ξ = 1.0, 5.0 and ε = 2.5 for the fluid with Pr = 0.01 is shown in Figure 3(a). The corresponding profiles for the temperature-dependent viscosity and thermal conductivity distributions are depicted in Figures 3(b) and 3(c). From Figure 3(a) it can be seen that an increase in the thermal conductivity-variation parameter, γ, leads to a rise in the velocity profile near the surface and at the boundary layer it decreases. Figures 3(b) and 3(c) show that viscosity and thermal conductivity increase with an the increase of the thermal conductivity variation parameter. For Pr = 0.01, ξ = 1.0, ε = 2.5 at η = 5.03870 the value of velocity increases by 18.12 per cent, 30.54 per cent and 41.52 per cent when γ increases from 0.0 to 1.0, 2.5 and 5.0 and the corresponding percentages for viscosity and thermal conductivity are 9.57 per cent, 15.03 per cent, 18.85 per cent and 78 per cent, 310 per cent and 540 per cent.

Conclusions

The effects of temperature-dependent viscosity and thermal conductivity on natural convection boundary layer flow about a truncated cone have been studied theoretically. The local non-similarity equations governing the flow in the truncated regime as well as far from the truncated cone are solved using the perturbation method. Numerical solutions to the equations governing the flow in the natural convection have also been obtained by the use of the implicit finite difference method. The comparison between the perturbation solutions obtained for two extreme regimes with the numerical solutions of finite difference method for the entire regime are found to be in excellent agreement. From the present investigation the following conclusions may be drawn:

(1) in the natural convection regime, both the local friction coefficient and the local Nusselt number decrease as the value of ε increases for all values of ξ;

(2) the local friction coefficient increases and the local Nusselt number decreases as the value of γ increases for all values of ξ;

(3) if the value of Pr increases, the local friction coefficient decreases and the local Nusselt number increases for the entire ξ regime;

(4) the velocity profiles increase and the corresponding viscosity and thermal conductivity of the fluid decrease owing to an increase in the value of ξ;

(5) when the value of the viscosity variation parameter, ε, increases the velocity profiles increase near the surface and the corresponding viscosity and thermal conductivity of the fluid decrease for all ξ;

(6) the velocity profiles as well as the corresponding viscosity and thermal conductivity of the fluid increase due to an increase in the value of γ; and

(7) the viscosity and the thermal conductivity of the fluid are approaches to unity at the outer edge of the boundary layer for values of all the pertinent parameters, which is expected.

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